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Bayes Estimators of Exponentiated Inverse Rayleigh Distribution using Lindleys Approximation

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Abstract

This work presents the results of a joint frequentist and bayesian approach to build Bayes estimators for the missing shape and scale parameters of the Exponentiated Inverse Rayleigh Distribution (EIRD). The shape and scale parameters of an EIRD were determined by applying the Bayes theorem to the posterior distribution. This distribution was tested under various loss functions, including entropy, linex, and scale invariant squared error, and was found to work with both conjugate and non-conjugate prior distributions. The shape and scale parameters' posterior distributions are difficult, so we used a Lindley approximation to get the ones we care about. Estimates for the size and shape parameters were obtained using the loss function, assuming that the scale and shape parameters are unknown and independent. Also the Bayes estimate for the simulated datasets and real life datasets were obtained. The Bayes estimates obtained under different loss functions are close to the true parameter value of the shape and scale parameters. The estimators are then compared in terms of their Mean Square Error (MSE) using R programming language. We deduce that the MSE reduces as the sample size (n) increases.

Keywords: Lindley's approximation; posterior distribution; prior distribution; entropy loss function; linex loss function; *scale invariant squared error loss function.*

1 Introduction

Rayleigh distribution originated from a two parameters Weibull distribution and it's a suitable model for mdistribution (IRD) was introduced by [1] for modeling realibility and survival data sets. [2] studied some properties of IRD and [3] discussed the properties and maximum likelihood estimation of the scale parameter of IRD. The variance and the higher order moments of this distribution do not exist. The reliability sampling plans of IRD was carried out by [4]. The probability density function (PDF) of the one parameter IRD The closed-form expressions for the mean, harmonic mean, geometric mean, mode and the median of IRD was discussed by [5]. The estimation of the parameter σ using both different classical and Bayesian estimation methods was carried out by [5] and [6]. In recent years, attention has been shifted to the generalization of probability distribution theory, most applied in reliability estimation [7, 8, 9, 10]. The transmuted Rayleigh distribution and transmuted generalized Rayleigh distribution were developed by [11, 12] respectively. [13] and [14] proposed a Beta Inverse Rayleigh. The exponentiated inverse

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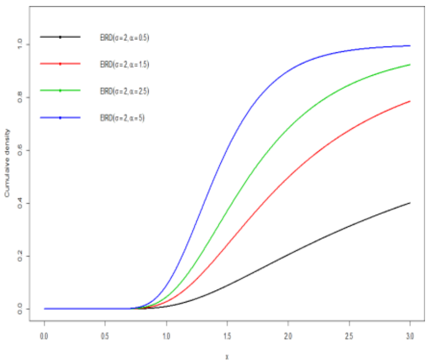
Vol-10 Issue-01 Jan 2022

Rayleigh distribution (EIRD) also known as a life time distribution was introduced by [15]. This distribution can be adopted for reliability estimation and statistical quality control. The probability density function (pdf) of EIRD is written as



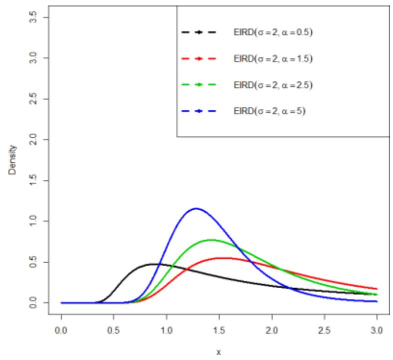


Fig. 1. PDF of an EIRD
Fig. 2. CDF of an EIRD

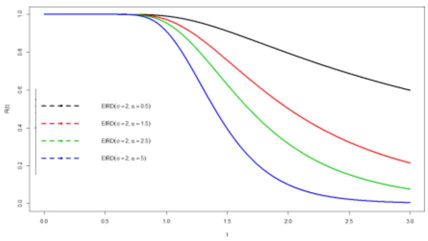


Fig. 3. Reliability graph of an EIRD

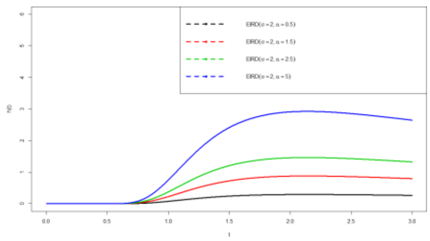


Fig. 4. Hazard graph of an EIRD

the Entropy Loss Function, Linex Loss Function and Scale Invariant Squared Error Loss Function given that the scale and shape parameters are unknown. In Sections 2, we discuss the estimation of the shape and scale parameters. In Section 3, numerical results are presented for both the simulated and real-life data on survival times of patients with breast cancer, and Section 4 contains the conclusion.

2 Materials and Methods

2.1 Maximum likelihood function

Let x = $(x_1, x_2, \ldots, x_n)$ be a random variable drawn from EIRD with size n. The likelihood function for the given random sample can be expressed as

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Vol-10 Issue-01 Jan 2022

$$L(x/\sigma, \alpha) = 2^n \sigma^n \alpha^n \prod_{i=1}^{n} x^{-3} e^{-\sum ((\frac{\sigma}{x})^2} \prod_{i=1}^{n} \left(1 - e^{-(\frac{\sigma}{x})^2}\right)^{\alpha-1} \tag{2.1}$$

Table 1. Estimates of the parameters of the four methods MLE, LLF, ELF and SISLF with their MLEs

n	Parameter		MLE		LLF		ELF		SISLF	
	σ	α	σ	α	σ	α	σ	α	σ	α
20	0.5 a=1.3 b = 1 k= 2	0.5	0.3277 (0.0708)	0.3037 (0.0759)	0.5298 (0.0155)	0.5417 (0.0269)	0.5280 (0.0114)	0.5320 (0.0272)	0.5212 (0.0105)	0.5195 (0.0273)
		1.2	0.5720 (0.0754)	1.5060 (0.4735)	0.5232 (0.0061)	1.2292 (0.2287)	0.5204 (0.0060)	1.3145 (0.2605)	0.5148 (0.0058)	1.3401 (0.3204)
		2	0.37725 (0.0561)	1.0128 (0.3028)`	0.5190 (0.0047)	2.1340 (0.8753)	0.516 (0.0047)	2.2064 (0.9440)	0.5119 (0.0045)	2.3326 (1.3331)
	1 a=0.5 b = 1.5 k= 0.5	2.5	0.8240 (0.1107)	1.6145 (0.5340)	1.0464 (0.0183)	2.8056 (1.6256)	1.0396 (0.0178)	2.7898 (1.7014)	1.0306 (0.0171)	3.0045 (2.5415)
		1	1.1475 (0.1800)	0.8599 (0.2520)	1.0582 (0.0310)	1.1127 (0.1654)	1.0495 (0.0272)	1.0791 (0.1579)	1.0371 (0.0257)	1.0870 (0.1874)
		1.5	1.1328 (0.1398)	2.1407 (0.7288)	1.0505 (0.0226)	1.6725 (0.4472)	1.0424 (0.0219)	1.6376 (0.4476)	1.0327 (0.0209)	1.6994 (0.5825)
50	0.5 a=1.3 b = 1 k= 2	0.5	0.4793 (0.0565)	0.4612 (0.0766)	0.5024 (0.0015)	0.5033 (0.0018)	0.5022 (0.0009)	0.5021 (0.0018)	0.5015 (0.0008)	0.50033 (0.0018)
		1.2	0.4947 (0.0452)	1.1250 (0.2167)	0.5086 (0.0021)	1.2271 (0.0060)	0.5075 (0.0021)	1.2341 (0.0638)	0.5051 (0.0021)	1.2374 (0.0677)
		2	0.4996 (0.0408)	1.9006 (0.4087)	0.5073 (0.0016)	2.0271 (0.2017)	0.5063 (0.0017)	2.0660 (0.2256)	0.5044 (0.0016)	2.1009 (0.2544)
	1 a=2.5 b = 1 k= 0.5	2.5	1.1066 (0.0817)	2.9527 (0.6750)	1.0179 (0.0064)	2.6076 (0.4002)	1.0151 (0.0063)	2.5940 (0.4056)	1.0115 (0.0062)	2.6571 (0.4714)
		1	1.1063 (0.0976)	1.2834 (0.2524)	1.0226 (0.0102)	1.0421 (0.044)	1.0190 (0.0096)	1.0276 (0.0427)	1.01401 (0.0094)	1.0251 (0.0444)
		1.5	1.0139 (0.0788)	1.8878 (0.3849)	1.0204 (0.0077)	1.5667 (0.1115)	1.0171 (0.0076)	1.5494 (0.1093)	1.0127 (0.0074)	1.5626 (0.1189)

Table 2. Estimates of the parameters of the four methods MLE, LLF, ELF and SISLF with their MLEs

n	Parameter		MLE		LLF		ELF		SISLF	
	σ	α	σ	α	σ	α	σ	α	σ	α
100	0.5 a=1.3 b = 1 k= 2	0.5	0.5862 (0.0453)	0.5640 (0.067)	0.5052 (0.0030)	0.5072 (0.0036)	0.5049 (0.0018)	0.5048 (0.0037)	0.5035 (0.0018)	0.5014 (0.0037)
		1.2	0.4559 (0.0281)	1.2014 (0.1609)	0.5041 (0.0010)	1.2129 (0.0281)	0.5035 (0.0010)	1.2159 (0.0291)	0.5023 (0.0010)	1.2166 (0.0298)
		2	0.4586 (0.0253)	1.9313 (0.2830)	0.5034 (0.0008)	2.0102 (0.094)	0.5029 (0.0008)	2.0303 (0.1003)	0.5020 (0.0008)	2.0457 (0.1061)
	1 a=2.5 b = 1 k= 0.5	2.5	1.0481 (0.0553)	2.3789 (0.3622)	1.0081 (0.0030)	2.5512 (0.1775)	1.0067 (0.0030)	2.5431 (0.1781)	1.0048 (0.0029)	2.5716 (0.1917)
		1	1.1096 (0.0714)	1.0723 (0.1424)	1.0108 (0.0045)	1.0205 (0.0197)	1.0090 (0.0044)	1.1096 (0.0192)	1.0065 (0.0044)	1.0117 (0.0196)
		1.5	0.9254 (0.0566)	1.2725 (0.1728)	1.0112 (0.0038)	1.5332 (0.0498)	1.0095 (0.0037)	1.5242 (0.0491)	1.0073 (0.0036)	1.5295 (0.0511)
200	0.5 a=1.3 b = 1 k= 2	0.5	0.4926 (0.0285)	0.4769 (0.0393)	0.5024 (0.0017)	0.5038 (0.0018)	0.5024 (0.0009)	0.5023 (0.0018)	0.5017 (0.0008)	0.5001 (0.0018)
		1.2	0.5350 (0.0230)	1.3687 (0.1349)	0.5018 (0.0005)	1.2051 (0.0135)	0.5015 (0.0005)	1.2065 (0.0137)	0.5009 (0.0005)	1.2066 (0.0139)
		2	0.5323 (0.0205)	2.3485 (0.2590)	0.5015 (0.0004)	2.0024 (0.0456)	0.5013 (0.0003)	2.0124 (0.0471)	0.5001 (0.0004)	2.0196 (0.0484)
	1 a=2.5 b = 1 k= 0.5	2.5	1.0183 (0.0374)	2.7745 (0.3128)	1.0041 (0.0015)	2.5210 (0.0818)	1.0034 (0.0015)	2.5172 (0.0819)	1.0025 (0.0015)	2.5296 (0.0845)
		1	1.053 (0.0474)	1.0707 (0.0995)	1.0054 (0.0022)	1.0093 (0.0092)	1.0044 (0.0021)	1.0056 (0.0091)	1.0032 (0.0022)	1.0047 (0.0091)
		1.5	1.0161 (0.0416)	1.6512 (0.1681)	1.0050 (0.0018	1.5162 (0.0242)	1.0042 (0.0018)	1.5117 (0.0240)	1.0031 (0.0018)	1.5140 (0.0244)

2.2 Application to Coating weight by chemical method on Tcs and

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Bcs.

In this section, the EIRD is applied to two (2) real data sets which were gotten from [15]. The first data set was a 72 observations on coating weight by chemical method on top center side (TCS) and the second data set was 72 observations on coating weight by chemical method on bottom center side (BCS).

For the Tcs data
36.8 47.2 35.6 36.7 55.8 58.7 42.3 37.8 55.4 45.2 31.8 48.3 45.3 48.5 52.8 45.4 49.8 48.2 54.5 50.1 48.4 44.2 41.2 47.2 39.1 40.7 40.3 41.2 30.4 42.8 38.9 34.0 33.2 56.8 52.6 40.5 40.6 45.8 58.9 28.7 37.3 36.8 40.2 58.2 59.2 42.8 46.3 61.2 58.4 38.5 34.2 41.3 42.6 43.1 42.3 54.2 44.9 42.8 47.1 38.9 42.8 29.4 32.7 40.1 33.2 31.6 36.2 33.6 32.9 34.5 33.7 39.9

For the Bcs
45.5 37.5 44.3 43.6 47.1 52.9 53.6 42.9 40.6 34.1 42.6 38.9 35.2 40.8 41.8 49.3 38.2 48.2 44.0 30.4 62.3 39.5 39.6 32.8 48.1 56.0 47.9 39.6 44.0 30.9 36.6 40.2 50.3 34.3 54.6 52.7 44.2 38.9 31.5 39.6 43.9 41.8 42.8 33.8 40.2 41.8 39.6 24.8 28.9 54.1 44.1 52.7 51.5 54.2 53.1 43.9 40.8 55.9 57.2 58.9 40.8 44.7 52.4 43.8 44.2 40.7 44.0 46.3 41.9 43.6 44.9 53.6 The data is summarized in Table 3.

Table 3. Estimates of the parameters of the four methods MLE, LLF, ELF and SISLF with their MLEs for the real life datasets

$a = 3, b = 4 \text{ and } k = 2$

Data	MLE		LLF		ELF		SISLF	
	σ	α	σ	α	σ	α	σ	α
Tcs	73.0125	13.1818	71.3211	12.2881	63.1867	12.9297	71.3211	12.9014
Bcs	78.5689	12.2331	76.9089	17.0602	69.1793	17.8291	70.1575	17.8466

Conclusion

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Vol-10 Issue-01 Jan 2022

In this work, we consider the classical method and Bayesian method under different loss functions such as Entropy Loss Function, Linex Loss Function and Scale Invariant Squared Error Loss Function. We employed the Bayesian techniques to obtain the posterior estimates of an EIRD using both conjugate and non-conjugate prior distribution under different loss functions and adopted the maximum likelihood approach to estimate the two parameter of interest. Fig. 1. shows that the PDF of the EIRD distribution at varying parameter values which shows that the distribution is positively skewed and the Fig. 2. is the CDF which shows the increasing pattern as other distributions. Fig. 3. shows the reliability graph which proves that the distribution can be used in lifetime studies since the graph tends to decrease as the time increases. Fig. 4. shows the hazard graph which shows the upside down bath-tub curve shape.

Table 1 and 2 shows the posterior estimates with MSE under different loss functions for the simulated datasets. Table 3, shows the posterior estimate on the real life dataset (coating weight by chemical method on top center side (TCS) and bottom center side (BCS))for different prior distribution under different loss functions

Based on the results displayed in Tables 1 and 2, we observed that all the posterior estimates for both shape and scale parameters for the simulated datasets are close to the true values of parameters of an EIRD. Also, we discovered the methods are consistent since the values of MSE decrease as sample size increases. It can be observed that the Bayesian estimates for both scale and shape parameters under the Bayesian techniques perform better than that of the classical techniques . The results obtained under the loss function ELF were quite more efficient than others loss functions because of its smallest MSE.

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Competing Interests

Authors have declared that no competing interests exist.

References

Trayer VN. Proceedings of the academy of science. Nauk, Belorus, U.S.S.R, Doklady Acad; 1964.

Iliescu DV, Voda V Gh. A study concerning the inverse rayleigh variate; 1972.

Voda VG. On the inverse Rayleigh distributed random variable. Rep. Statist. App. Res., Juse. 1972;19:13-21.

Rosaiah K, Kantam RRL. Acceptance sampling based on the inverse rayleigh distribution. Economic Quality Control. 2005;20(2):277-286.

Gharraph M. Comparison of estimators of location measures of an inverse Rayleigh distribution. The Egyptian Statistical Journal. 1997;37:295-309.

Soliman A, Amin EA, Abd-El Aziz AA.

ISSN: 1832-5505
Vol-10 Issue-01 Jan 2022

Estimation and prediction from inverse Rayleigh distribution based on lower record values. Applied Mathematical Sciences. 2010;4(62):3057- 3066.

Mudholkar GS, Srivastava DK. Exponentiated Weibull family for analyzing bathtub failure- rate data. IEEE Transactions on Reliability. 1993;42(2)299-302.

Mudholkar GS, Srivastava DK, Friemer M. The exponentiated weibull family: a reanalysis of the bus-motorfailure data. Technometrics. 1995;37(4):436-445.

Gupta RC, Gupta PL, Gupta RD. Modeling failure time data by Lehman alternatives. Communications in Statistics - Theory and Method. 1998;27(4):887-904.

Nadarajah S, Kotz S. The exponentiated type distributions. Acta Applicandae Mathematicae. 2006;92(2):97-111.

Merovci F. Transmuted rayleigh distribution. Austrian Journal of Statistics. 2013;42(1):21-31.

Merovci F. Transmuted generalized Rayleigh distribution. Journal of Statistics Applications and Probability. 2014;3(1):9-12.

Ahmad A, Ahmad S, Ahmed A. Transmuted inverse rayleigh distribution: A generalization of the inverse rayleigh Distribution. Mathematical Theory and Modeling. 2014;4(7):90-98.

Leao J, Saulo H, Bourguignon M, Cintra R, R-lgo L, Cordeiro G. On some properties of the beta Inverse Rayleigh distribution. Chilean Journal of Statistics. 2013;4(2):111-131.

Rao, Gadde Srinivasa, Mbwambo, Sauda. Exponentiated inverse rayleigh distribution and an application to coating weights of iron sheets data. Hindawi Journal of Probability and Statistics. 2019;1-13.

Obisesan KO, Adegoke TM, Adekanmbi DB, Lawal M. Numerical approximation to intractable likelihood functions. Perspectives and Developments in Mathematics. 2015;301-324.

Yahaya AM, Dibal NP, Bakari HR, Adegoke TM. Obtaining parameter estimate from the truncated poisson probability distribution. North Asian International Research Journal of Sciences, Engineering & I.T. 2016;2(9):3-10.

Bakari HR, Adegoke TM, Yahya AM. Application of Newton Raphson method to non-linear models. International Journal of Mathematics and Statistics Studies. 2016;4(4):21-31.

Varian HR. A Bayesian approach to real estate assessment. North Holland, Amsterdam. 1975;195 -208.

Nassar MM, Eissa FH. Bayesian estimation for the exponentiated Weibull model. Commun Stat Theory Methods. 2004;2343-2362.

Soliman AA. Estimation of parameters of life from progressively censored data using Burr XII model. IEEE Trans. Reliability. 2005;54(1):34-42.

Soliman AA, Abd Ellah AH, Sultan KS. Comparison of estimates using record statistics from Weibull model: Bayesian and non-Bayesian Approaches. Computational Statistics & Data Analysis. 2006;51:2065-2077.

Pandey BN. Estimator of the scale parameter of the exponential distribution using LINEX loss function. Commun. Statist. Theory Meth. 1997;26;(9):2191-2202.

Basu AP, Ebrahimi N. Bayesian approach to life testing and reliability estimation using asymmetric loss function. J. Statist. Plann. Inference.

ISSN: 1832-5505
Vol-10 Issue-01 Jan 2022

1991;29:21-31.
Rojo J. On the admissibility of with respect to the LINEX loss function. Commun. Statist.- Theory Meth. 1987;16(12):3745-3748.
Preda Vasile, Panaitescu Eugenia. Bayes estimators of Modified-Weibull distribution parameters using Lindley's approximation. WSEAS Transactions on Mathematics. 2010;9(7):539-549.
Calabria R, Pulcini G. An engineering approach to Bayes estimation for the Weibull distribution. Microelectronics Reliability. 1994;34(5):789-802.

Dey DK, Ghosh M, Srinivasan C. Simultaneous estimation of parameters under entropy loss.
J. Statist. Plan. and Infer. 1987;347-363.
Dey DK, Liu Pie-San L. On comparison of estimators in a generalized life model. Micro- electron. Reliab. 1992;45(3):207-221.
Sule BO, Adegoke TM. Baysian approach in estimation of shape parameter of an Lindley DV. Approximate bayes methods. Bayesian statistics. Valency; 1980.
Dibal NP, Adegoke TM, Yahaya AM. Bayes' estimators of an exponentially distributed random variables using al-bayyati's loss function. International Journal of Scientific and Research Publications. 2019;9(12):674-684.
Adegoke TM, Nasiri P, Yahya WB, Adegoke GK, Afolayan RB, Yahaya AM. Bayesian estimation of Kumaraswamy distribution under different loss functions. Annals of Statistical Theory and Applications. 2019;2:90-102.
Dodge, Yadolah. The concise encyclopedia of statistics. Spinger Science & Business Media; 2008.
exponential inverse exponential distribution. Asian Journal of Probability and Statistics. 2020;9(1):13-27.
Adegoke TM, Yahya WB, Adegoke GK. Inverted generalized exponential. Annals of Statistical Theory and Applications. 2018;1:1-10.
DeRoot MH. Optimal statistical decision. New York. McGraw-Hill; 1970.
Awad Manahel, Rasheed Huda A. Bayesian and non - bayesian inference for shape parameter and reliability function of basic gompertz distribution. Baghdad Science Journal. 2020;17(3):854-860.
Gupta Isha. Estimation of parameter and reliability function of exponentiated inverted weibull distribution using classical and bayesian approach. International Journal of Recent Scientific Research. 2017;8(7):18117-18819.
Zellner A. Bayesian estimation and prediction using asymmetric loss functions. Jour. Amer. Statist. Assoc. 1986;446-451.